Warm-Up (Part 1)

Chapter 11 Formulas

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$a_n = a_1 + (n-1)d$$

$$a_n = a_1 \cdot r^{n-1}$$

1. 8, 13, 18, 23…

   Find the sum of the first 47 terms

$a_{47} = 238$

$S_{47} = 5,781$

2. If you pick one card from a standard deck, what is the probability that it is a club or a numbered card?

$$P(A) + P(B) - P(A \cap B)$$

$$\frac{13}{52} + \frac{36}{52} - \frac{9}{52} = \frac{40}{52} = \frac{10}{13}$$

Example 1: If you draw two cards from a standard deck what is the probability that both cards will be Jacks?

(WITH REPLACEMENT) $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

(WITHOUT REPLACEMENT) $\frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

Example 2: What is the probability of rolling a 5 three times in a row given a standard die?

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

Section 12.1 Counting Principles

Objectives

To determine how many ways an event can occur (in a variety of situations).

Methods

- Fundamental Counting Principle
- Permutations
- Combinations

Counting Principle

If one event can be done **x** number of ways and another event can be done **y** number of ways, then both can be chosen by **multiplying x and y**

Example 1: You are buying a Honda. You can either get an Accord or a Civic. Your options for color are red, black, or silver. Your options for the interior color are black, gray, and tan. How many options do you have?

$2 \cdot 3 \cdot 3 = 18$

Fundamental Counting Principle

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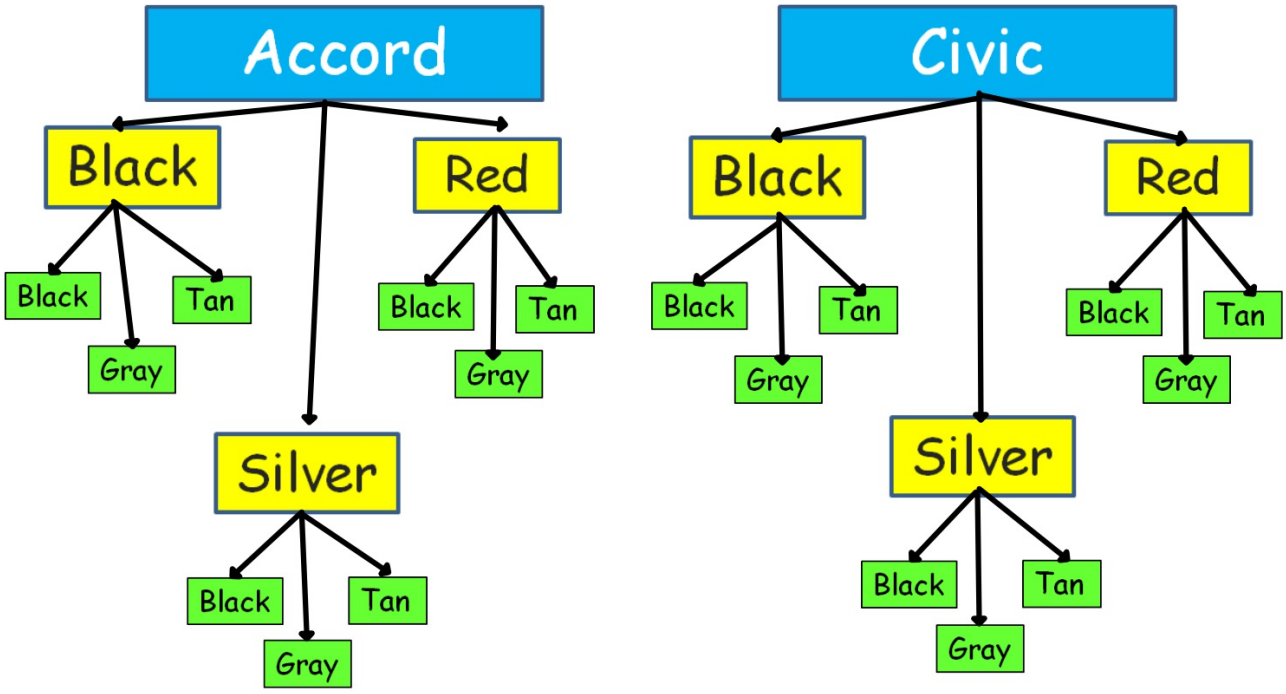
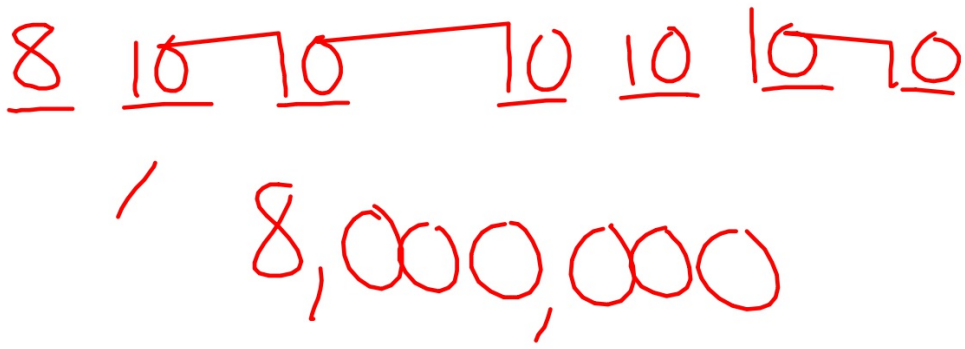


Example 2: How many different 7-digit phone numbers exist in the 215 area code? (Note: Local telephone numbers cannot begin with 0 or 1).



Example 3: A college student is preparing a course schedule for next semester. The student must take four courses total (one math, one literature, one science, and one social science) and has the following choices:

5 Math, 4 Literature, 3 Sciences, 8 Social Sciences

How many schedules are possible?

4) How many ways can 8 members of a family be seated side-by-side in a movie theater if the father is seated on the aisle?

1 7 6 5 4 3 2 1

5,040

5) A photographer is taking a picture of a bride and a groom together with 6 attendants. How many ways can he arrange the 8 people in a line if the bride and groom stand in the middle?

6 5 4 2 1 3 2 1

1,440

Practice: Solve each problem.

1) The letters A, B, C, and D are used to form four-letter passwords. How many passwords are possible if the letters can be repeated any number of times?

4 4 4 4 = 256 passwords

2) A restaurant serves 5 entrees, 3 salads and 4 desserts. How many different meals could be ordered if each has an entrée, a salad, and a dessert?

60 meals

3) How many 5-digit even numbers can be formed using the digits 2, 4, 6, 7, 8, if the digits can be repeated any number of times?

5 5 5 5 4 = 2,500 numbers

4) How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed?

26 26 26 10 10 10 = 17,576,000

5) A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible?

10 10 10 = 1,000

Permutations and Combinations

Permutations: Choosing/Arranging items from a group- Order Matters!!

Combination: Choosing items from a group- Order Does Not Matter!

Permutations

Notation: ${}_nP_r$

n: number of objects you are choosing from

r: number of objects you are putting in order

Formula

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

Don't need to memorize

! - Factorial

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Example 1: How many ways can 1st, 2nd, and 3rd prizes be awarded from a set of six finalists?

$$_6P_3 = \frac{n!}{(n-r)!} = \frac{6!}{(6-3)!} = \frac{6!}{3!}$$

Example 2: You have 13 books. You have room for 6 books on your shelf. How many ways can you select and arrange 6 books?

Permutation with Repetition:

The number of permutations on n objects of which p are alike, q are alike, etc... is

$$\frac{n!}{p!q!\ldots}$$

Permutation with Repetition: The number of permutations on n objects of which p are alike, q are alike, etc... is $\frac{n!}{p!q!\ldots}$

Example 1: How many 6-letter patterns can be formed from the letters of the word COTTON?

$$\frac{6!}{2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 180$$

Example 2: How many ways can 11-letter patterns can be formed from the letters of the word MISSISSIPPI?

$$\frac{11!}{4!4!2!} = 11!/(4!4!2!)$$

34,650 patterns

Practice: Solve each problem.

5) How many different ways can the first five letters of the alphabet be arranged if each is used only once?

5P5

5 4 3 2 1

120

6) How many different ways can the 4 call letters of a radio station be arranged if the first letter must be W or K and no letters repeat?

2 25 24 23

27,600

7) How many ways can 3 books be arranged on a shelf if chosen from a selection of 7 different books?

7P3

210 ways

8) How many different ways can 4 different books be arranged?

4 3 2 1

24

Practice: Solve each problem.

9) How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1 and no digit can be repeated?

10) How many 5-digit numbers can be made using the digits from 76,627?